

Flexible time series modelling with SSpace

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Introduction

SSpace is a package that implements linear, non-linear and non-Gaussian State Space systems in a very flexible and powerful way.

It is mainly based on the work of Peter C Young and collaborators with other elements, like the book of Durbin and Koopman (2012), resembling many well-known alternative packages, but there are some unique interesting elements in it...

Introduction

Statistical features:

- 1 The statistical framework is very general in the equations formulation and in the sense that all system matrices are multivariate and potentially time-varying.
- 2 Exact or diffuse initialisation of Kalman Filtering and state and disturbance smoothing.
- 3 Use of exact gradient in Maximum Likelihood estimation, when possible.
- 4 Kalman filtering and smoothing based on univariate treatment of multivariate systems.
- 5 Other estimation procedures, i.e. Concentrated Likelihood and minimum RMSE of several steps ahead are implemented.
- 6 ...

Introduction

Other advantages:

- 1 It is free.
- 2 Simple in its usage, since models are standard functions written in MATLAB.
- 3 A wide range of templates for specific models are available (Unobserved Components, ARIMA, Regression, Exponential Smoothing, Stochastic Volatility, etc.).
- 4 A full time series analysis may be done with a reduced number of coded functions with names that have been carefully selected following nemotechnic rules so that the user may remember them easily.
- 5 Minimizer may be chosen (fminunc).
- 6 We have a fully MATLAB version, we have coded core functions in C++ and finishing a R version.

Linear-Gaussian State Space system

State equation:

$$\alpha_{t+1} = T_t \alpha_t + \gamma_t + R_t \eta_t$$

Observation equation:

$$y_t = Z_t \alpha_t + d_t + C_t \epsilon_t$$

- η_t : vector of state noises $v \times 1$, $N(0; Q_t)$
- ϵ_t : vector of observed noises $h \times 1$, $N(0; H_t)$
- $S_t = E(\epsilon_t, \eta_t')$ covariance matrix $h \times v$
- $\gamma_t = f(\Gamma_t, u_t)$ and $d_t = g(D_t, u_t)$, with u_t of dimensions $k \times 1$
- y_t : output vector $m \times 1$
- α_t : state vector $n \times 1$
- Every term is potentially multivariate and time-varying
- Many different ways to write the same system

Non-Gaussian State Space system

State equation:

$$\alpha_{t+1} = T_t \alpha_t + \gamma_t + R_t \eta_t$$

Observation equation:

$$y_t \sim p(y_t | \theta_t) + d_t$$

$$\theta_t = Z_t \alpha_t$$

- 1 Exponential family distribution, where $p(y_t | \theta_t) = \exp[y_t' \theta_t - b_t(\theta_t) + c_t(y_t)]$, $-\infty < \theta_t < \infty$.
- 2 Stochastic Volatility models, i.e. $y_t = \exp(\frac{1}{2} \theta_t) \epsilon_t + d_t$.
- 3 Observations generated by the relation $y_t = \theta_t + \epsilon_t$, $\epsilon_t \sim p(\epsilon_t)$, with $p(\bullet)$ being a distribution of the exponential family.

Non-Linear State Space system

State equation:

$$\alpha_{t+1} = T_t(\alpha_t) + \gamma_t + R_t(\alpha_t)\eta_t$$

Observation equation:

$$y_t = Z_t(\alpha_t) + d_t + C_t(\alpha_t)\epsilon_t$$

$$\eta_t \sim N(0, Q_t(\alpha_t)), \epsilon_t \sim N(0, H_t(\alpha_t))$$

The system is actually non-linear and/or state-dependent. At the moment the Extended Kalman Filter and state smoothing with exact initialisation are the implemented algorithms (Koopman and Lee, 2009).

1. Specify the model

For a local level model:

- State equation:
- Observation equation:

$$L_{t+1} = L_t + \eta_t$$

$$y_t = L_t + \epsilon_t$$

We have to fit that model in the general framework:

- State equation:
- Observation equation:

$$\alpha_{t+1} = T_t \alpha_t + \gamma_t + R_t \eta_t$$

$$y_t = Z_t \alpha_t + d_t + C_t \epsilon_t$$

$\alpha_t = L_t$; $T_t = 1$; $R_t = 1$; $Z_t = 1$; $C_t = 1$; γ_t and d_t do not exist.

$S_t = 0$ and Q_t and H_t are two unknown scalar values that have to be positive

2. Translate

Translate model to a MATLAB function (e.g. using templates).

```
1 model= SampleSS(p)
2 model.T= []; model.Gam= []; model.R= []; model.Z= []; model.D= [];
3 model.C= []; model.Q= []; model.H= []; model.S= [];
```

For the local level model: $L_{t+1} = L_t + \eta_t$ $y_t = L_t + \epsilon_t$

```
1 model= example1(p)
2 model.T= 1; model.Gam= []; model.R= 1; model.Z= 1; model.D= [];
3 model.C= 1; model.Q= 10.^p(1); % This makes variance always positive
4 model.H= 10.^p(2); model.S= 0;
```

3. Set up the model

```
1 sys= SSmodel('y', y, 'model', @example1);
```

Using duplets it is possible to select the input and output data, the model filename, additional inputs to the function that implements the model, initial values for parameters, fix values for parameters, diffuse or exact initialisation of algorithms, objective cost function and analytical or exact score in Maximum Likelihood estimation.

3. Set up the model

Inputs to the system (and to SSmatrix):

```

1           y: [1x100 double]
2           u: []
3           model: @example1
4   user_inputs: {}
5           p0: [2x1 double]
6           p: [2x1 double]
7           a1: NaN
8           P1: NaN
9   OBJ_FUNCTION_NAME: {@llik}
10          gradient: 1
11          Nsimul: 300

```

3. Set up the model

Outputs of operations on models:

```
1      table: []  
2      obj_value: 0  
3      llik: 0  
4      covp: []  
5      mat: [1x1 struct]  
6          a: 740.0149  
7          P: 0.7781  
8      yfit: []  
9          F: []  
10     yfor: []  
11     Ffor: []  
12     v: []  
13     Fv: []  
14     eta: []  
15     eps: []  
16     Veta: []  
17     Veps: []
```

4. Estimation, validation, filtering, ...

```
1 sys= SSestim(sys);  
2 sys= SSvalidate(sys);
```

```
1 sys= SSfilter(sys); % Just for filtering estimates  
2 sys= SSsmooth(sys); % For state smoothing estimates  
3 sys= SSdisturb(sys); % If disturbance smoothing is required
```

Results are stored in sys output structure. It stores parameters with covariance matrix, optimal states and covariances, fitted output values and covariances, forecasted values and covariances, innovations, disturbances estimates with covariances. Further statistical diagnostics are advisable.

Example 1: ARIMA and eXogenous terms

The standard ARIMA template is:

```

1 model= SampleARIMA(p)
2 Sigma = ; % Covariance matrix of perturbation
3 DIFFpoly= ; % Difference operator of output(s)
4 ARpoly = ; % (V)AR polynomial
5 MAPoly = ; % (V)MA polynomial
6 D= ; % Inputs
    
```

```

1 model= SampleARIMA(p)
2 Sigma = 10.^p(1); % Covariance matrix of perturbation
3 DIFFpoly= 1; % Difference operator of output(s)
4 ARpoly = [1 p(2) p(3)]'; % (V)AR polynomial
5 MAPoly = [1 p(4)]'; % (V)MA polynomial
6 D= p(5); % Inputs
    
```

Example 1: ARIMA and eXogenous terms

```

1 model= SampleARIMA(p, u)
2 Sigma = 10.^p(1); % Covariance matrix of perturbation
3 DIFFpoly= [1 -1]'; % Difference operator of output(s)
4 ARpoly = conv([1 p(2) p(3)]', [1 zeros(1, 11) p(5)]'); % (V)AR
5 MApoly = [1 p(4) -p(4)]'; % (V)MA polynomial
6 D= p(5)*u; % Inputs (row vector)

```

```

1 ...
2 D= filter(1, [1 -p(5)], u)'; % Inputs

```

```

1 ...
2 D= 1./(exp(-p(5)*u)); % Inputs

```


Example 1: ARIMA and eXogenous terms

```

1 model= SampleARIMA(p, u, y)
2 ...
3 D= p(5)*u; % Inputs
4 ind= find(y> 0);
5 D(1, ind)= p(6)*u(1, ind);

```

Example 2: Unobserved Components (DHR)

```

1 function model= SampleDHR(p, N)
2 m= ;
3 % Trend model
4 I= eye(m); O= zeros(m);
5 TT = [I I;0 I]; ZT = [I O]; RT = [I O; 0 I];
6 QT = blkdiag(varmatrix(), varmatrix());
7
8 % Seasonal/cyclical DHR components
9 Periods = ;
10 Rho = ;
11 Qs = repmat(varmatrix(), [], []);
12
13 % Linear Regression effects
14 D= [];
15
16 % IRREGULAR COMPONENT (often empty if VAR component is included)
17 H = varmatrix();

```

Example 2: Unobserved Components (DHR)

```

1 function model= SampleDHR(p, N)
2 m= 3;
3 % Trend model
4 I= eye(m); O= zeros(m);
5 TT = [I I;0 I]; ZT = [I 0]; RT = [I 0; 0 I];
6 QT = blkdiag(varmatrix(p(1:6)), varmatrix(p(7:12)));
7
8 % Seasonal/cyclical DHR components
9 Periods = repmat([4 2], 3, 1);
10 Rho = ones(3, 2);
11 Qs = repmat(varmatrix(p(19:24)), 1, 2);
12
13 % Linear Regression effects
14 D= [];
15
16 % IRREGULAR COMPONENT (often empty if VAR component is included)
17 H = varmatrix(p(13:18));

```

Example 2: Variance of seasonal components

```

1  ...
2  Qs= [varmatrix(p(19:24)) varmatrix(p(19:24))];
3  Qs= [varmatrix(p(19:24)) varmatrix(p(25:30))];
4  Qs= [diag(10.^p(19:21)) diag(10.^p(19:21))];
5
6  % Next code produces a time varying covariance matrix with a sudden
   change in observation 100
7  ...
8  Hc1= varmatrix(p(13:18));
9  H= repmat(Hc1, [1 1 N]);
10 Hc2= varmatrix(p(25:30));
11 H(:, :, 100:N)= repmat(Hc2, [1 1 N-99]);

```

Example 3: Comparison with other software

(Airline data example of Box and Jenkins)

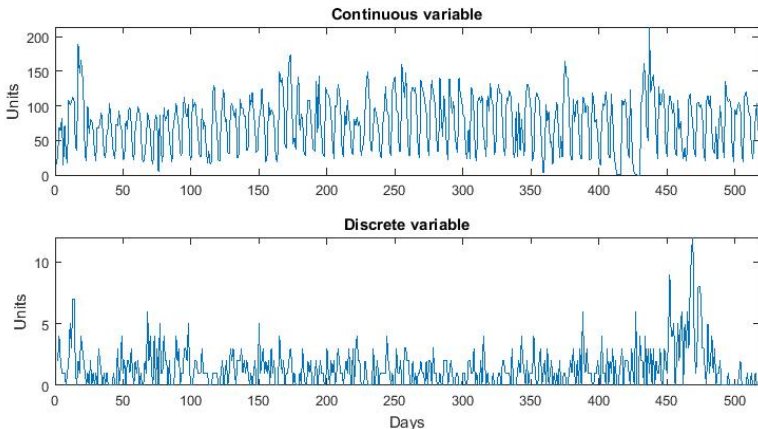
(All variances are $1e-6$)

Variances	STAMP	SSpace	Variance	SsfPack	SSpace
Trend	297.84	297.75	Trend	238	237.97
Slope	0.00	0.00	Slope	0	0.00
Seas	3.55	3.55	Seas1	11	11.11
Irregular	237.93	237.82	Seas2	5	5.26
			Seas3	0	0.00
			Seas4	2	2.30
			SEas5	1	1.23
			Seas6	0	0.00
			Irr.	327	329.78
LogLik	413.82	216.08	LogLik	223.46	223.27

Example 4: Forecast comparisons

- Daily data on sales demand of 261 products of a retailer in Spain
- Absolute error of 90 runs of 14 days ahead with daily data
- 36 % of time series are modeled as poisson processes
- Automatic identification techniques based on BIC:
 - NAIVE: random walk
 - AR up to 32th order
 - ARIMA (Hyndman & Khandakar, 2008)
 - Exponential Smoothing (ES)
 - Unobserved Components (UC)
 - Poisson Unobserved Components (UCp)
 - Mean combination of ARIMA, ES, UC and UCp
 - Median combination of ARIMA, ES, UC and UCp

Example 4: Forecast comparisons



Forecast comparisons

Table for continuous time series (168 series):

	1	2	3	4	7	14
NAIVE	0.4695	1.0833	1.7437	2.4153	4.0597	8.3784
AR	0.3484	0.7198	1.0967	1.4816	2.6518	5.5116
ARIMA	0.3210	0.6634	1.0133	1.3695	2.4550	5.0598
ES	0.3308	0.6849	1.0455	1.4117	2.5241	5.2015
UC	0.3164	0.6539	0.9989	1.3500	2.4225	4.9901
Mean	0.3209	0.6635	1.0127	1.3691	2.4563	5.0740
Median	0.3216	0.6655	1.0163	1.3728	2.4590	5.0774

Example 4: Forecast comparisons

Table for Discrete series (93 series):

	1	2	3	4	7	14
NAIVE	1.0128	2.0892	3.1674	4.2484	7.3912	14.9254
AR	0.8304	1.6736	2.5218	3.3707	5.9269	11.9996
ARIMA	0.8285	1.6634	2.5020	3.3425	5.8658	11.8557
ES	0.8338	1.6762	2.5220	3.3694	5.9136	11.9533
UC	0.8147	1.6331	2.4542	3.2784	5.7589	11.6495
Poisson	0.8056	1.6161	2.4284	3.2456	5.7119	11.5940
Mean	0.8089	1.6242	2.4428	3.2642	5.7368	11.6101
Median	0.8104	1.6259	2.4445	3.2660	5.7378	11.6103

Conclusion

- SSpace is a toolbox for full SS system analysis
- Ask the authors for availability
- Documentation and forum in solid-analytics.org
- You may 'buy' it by pieces:
 - Just filtering, smoothing and the general template (SSfilter, SSsample)
 - Previous and additional templates
 - Part of the previous and optimization

Thank you for your attention!

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