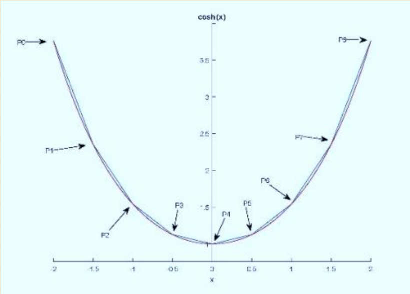

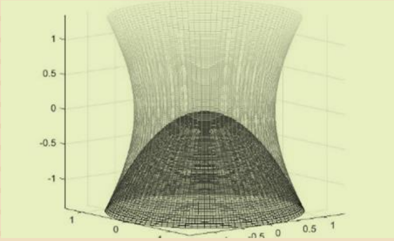

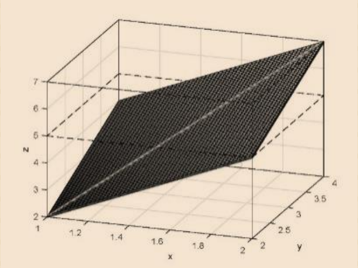

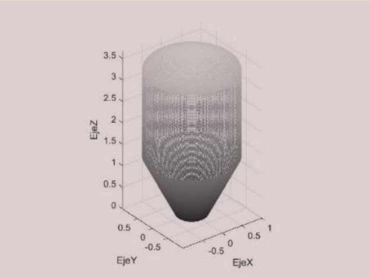

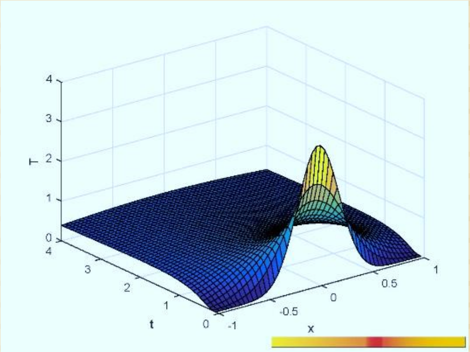

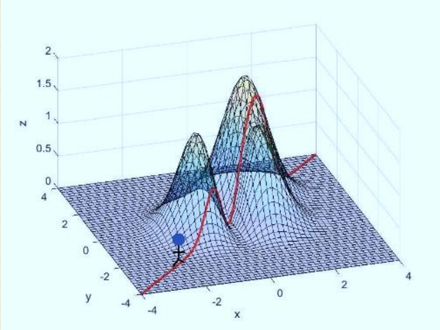

ENSEÑANZA DE CÁLCULO DIFERENCIAL E INTEGRAL A PARTIR DE PROBLEMAS APLICADOS

Cristina Solares, Henar Herrero, Rocío Blanco

Introduction

- Calculus concepts are abstract and difficult for students.
- We have found that some students show very little understanding of the calculus concepts.
- The Necessity principle (Harel, 1998) states that"…for students to learn, they must see an intellectual need (as opposed to social or economic) for what they are intended to be taught".
- In this study we propose to introduce the concepts starting from applied contexts, providing to the students problems related to their studies.
- Students participate actively and the new concepts involved become meaningful for them.
- We respect the process of abstraction, from particular to general.

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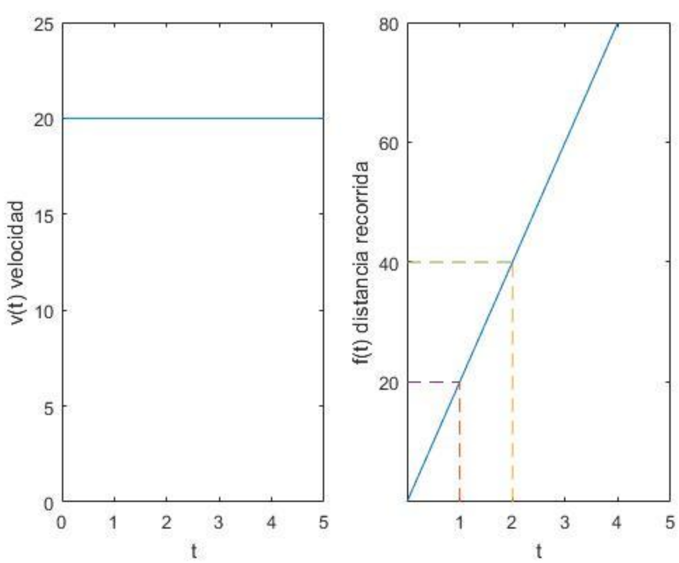
Calculus I: Derivative

Velocity and Distance Traveled by a Vehicle

A vehicle moves, on a straight line, at a constant velocity of v= 20 km / h. The distance traveled by the vehicle after 1 h is 20 km and after 2 h it is 40 km (Strang, 2010).



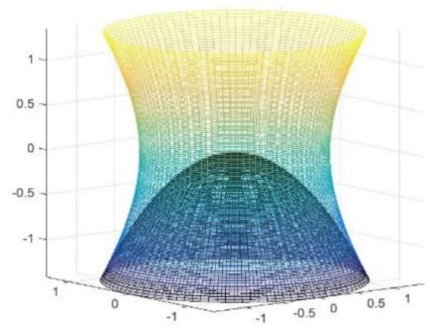
Question 1: *What is the distance traveled by the vehicle after t hours?* The students answer that the distance traveled is f (t) = v t= 20 t.

Question 2: If we study the displacement of the vehicle in the time interval t∈ [1,2], we see that at t = 1 h the vehicle has traveled 20 km and at the instant t = 2 h the vehicle has traveled 40 km. *What is the increase in time? What is the increase in displacement?* The students answer that the increase in time is 1 h and the increase in the displacement in that interval is 20 km.

The concept of *increasing of a variable and increasing of a real function* is introduced.

Volume of a Water Tank

We have worked with a water tank
Hipódromo de la Zarzuela, E. Torroja, 1935 (Barrio, 1994)



- The exterior enclosure (Part I) is formed by a hyperboloid of revolution of equation:

$$x^2 + y^2 - \frac{z^2}{2} = 1, z \in \left[-\sqrt{2}, \sqrt{2}\right].$$

- The tank bottom (Part II) is formed by a paraboloid of equation:

$$z = -\frac{x^2 + y^2}{\sqrt{2}}, z \in \left[-\sqrt{2}, 0\right].$$

- The water is stored above the paraboloid and inside the hyperboloid.
- The maximum and minimum levels of the tank are,

$$z_{min} = -\sqrt{2},\ z_{max} = \sqrt{2}$$

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Calculus I: Definite Integral

Volume of a Water Tank

The teacher explains how we can fill the tank with disks perpendicular to the axis of rotation, whose volume is known. These disks are obtained by taking different partitions of the interval of variation of the variable. Adding the volumes of all the discs, we can approximate the total volumen as $\sum_{1=1}^{n} \pi\, x(c_i)^2\, \Delta y_i$, where n is the number of subintervals in the partition and c_i is the middle point and Δy_i is the length of the ith subinterval.

Question 1: *Calculate the volume of the Part I with n=4 and n=10 discs.* The students answer that for n=4 the volume is 11.8014 and for n=10 the volume is 11.8403.

Question 2: *Calculate the volume of the Part II with n=4 and n=10 discs.* The students answer that for n=4 the volume is 4.4429 and for n=10 the volume is 4.4429.

At this point we introduce the definite integral that will be used to calculate the *volume of a solid of revolution* and the teacher solves the above problem. The exact volumen is 7.4048.

The students work with another applied problem and calculate the capacity of a silo answering to different questions.

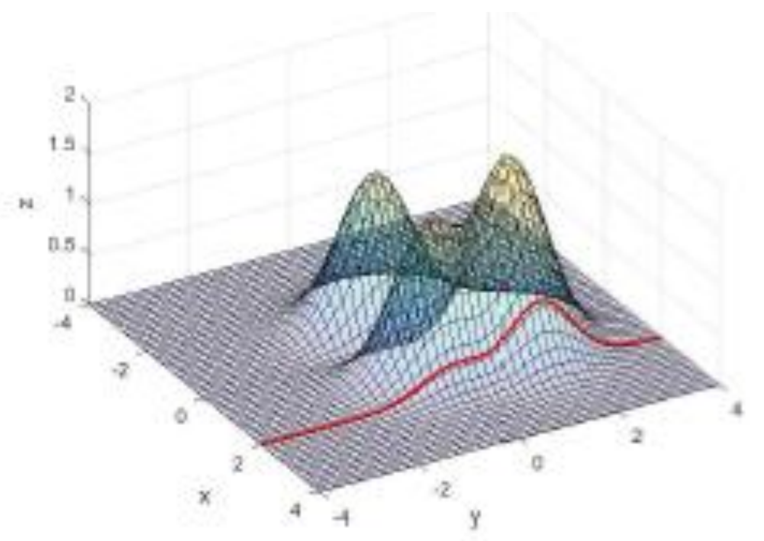
Calculus II: Partial and Directional Derivatives

Mountain Problem

Consider the following function, whose graph represents a mountain (Cooper, 2001)

$$z = f(x,y) = 4\,x^2 e^{-x^2-y^2} + y^2 e^{-(x-1)^2-(y-1)^2}$$

with $x,y \in [-4,4]$.



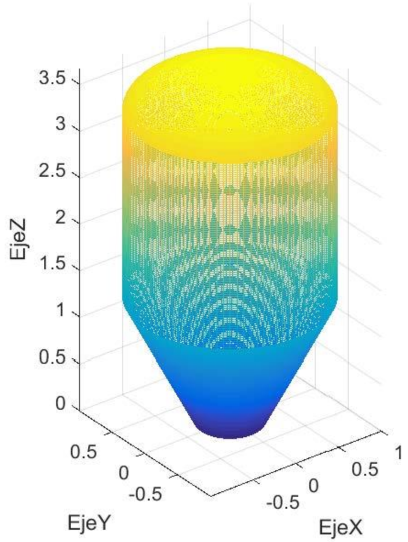
Question 1: *What does f (2, 2) = 0.5467 mean?* The students answer that it is the height of the mountain at the point (x,y)=(2,2) in the plane.

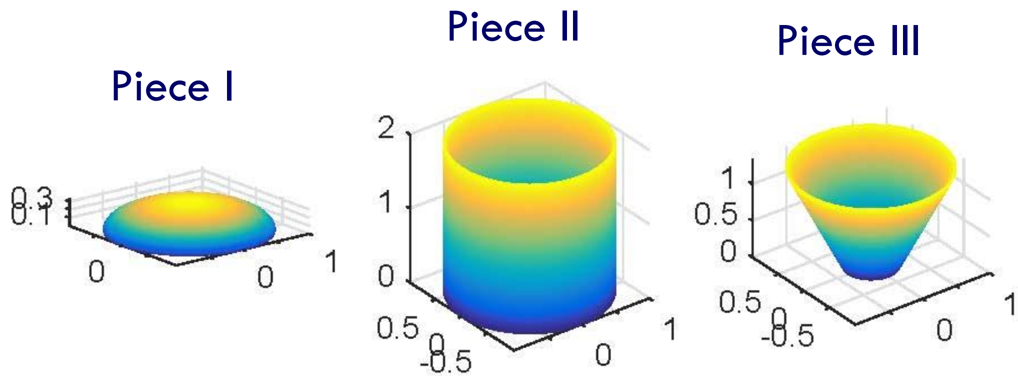
Question 2: *What is the meaning of g = f (2,t), for $t \in [-4,4]$?* The students answer that it is a function of t that gives the height on the mountain of a mountaineer who is at a point lying above (x, y) = (2,-4) and moves so that his horizontal displacement is in the direction **j** = (0,1) .

At this point the teacher draws the curve $\{x = 2, y = t,\ z = f(2,t)\}, t \in [-4,4]$, over the surface.

Silo Problem

We have worked with a metal silo (Barrio, 1994)





The equation of piece I is $x^2 + y^2 + 9\,z^2 = 1, z \in [0, 1/3]$.
The equation of the intermediate piece II is $x^2 + y^2 = 1, z \in [0,2]$.
The equation of piece III (lower part) is

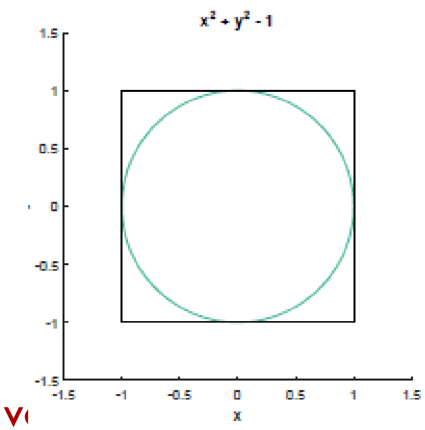
$$x^2 + y^2 - \left(\frac{z}{2} + \frac{1}{3}\right)^2 = 0, z \in [0, 4/3].$$

Question: *Calculate the volume of the silo using double integrals.*

Question 1: *Draw schematically each of the pieces, supporting them in the $z = 0$ plane.* The students draw each piece of the silo.

Question 2: The projection of piece I on the plane $z = 0$ is the integration domain D_I. *Draw the projection and write the equation of the curve enclosing the domain* D_I. The students work in the question and the teacher shows the solution.

Question 3: The maximum height of the piece I is 1/3, we could calculate the volume of that part as the volume of a prism with base
$R = \{(x, y) \ / \ x \in [-1,1], y \in [-1,1]\}$ and height 1/3.



Calculate the volume of that prism. The students answer that the v .33 and the teacher explains that the real volume of the piece I is 0.70.

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